Structural Damage Detection, Locating, and Quantifying Using Dynamic Data *

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Abstract— **This paper deals with a methodology for the use of dynamic response as an inspection and surveillance tool for the damage in a structure. The method is based on finite element discretisation to identify the stiffness characteristics (related to cracking) starting from modal dynamic parameters (natural frequency, mode shape, and damping) derived from dynamic tests. Any damage in the structure alters its dynamic characters. The damage reduces the stiffness of the structure and increases its damping value, at the same time it will decrease the natural frequency and the corresponding mode shape changes. A three stage method was proposed to identify, locate, and quantify the extent of damage. In order to locate the damaged structure Modal Assurance Criterion (MAC), Co-ordinate Modal Assurance Criterion (COMAC), Normalized Modal Difference (NMD), and Direct Natural Frequency Correlation were used in the first stage. In the second stage Curvature Damage Factor (CDF) using curvature mode shape was used to locate the damaged positions. Neural Network was introduced in the final stage to determine the intensity of damage. Numerical results show the high efficiency of the proposed method for accurately identifying, locating, and extent of multiple structural damages.**

Keywords— ***Modal assurance criterion; Co-ordinate modal assurance criterion; normalized modal difference; natural frequency correlation; curvature damage factor; neural network***

I. INTRODUCTION

Normally design of civil infrastructures such as buildings, bridges etc should have long life span. Changes in load characteristics, deterioration with age, environmental influences and random actions may cause local or the whole damage to the structures. A continuous health monitoring of structures will enable the early identification of damage and allow appropriate retrofitting to prevent potential sudden structural failures. In recent years, the damage assessment of structures has drawn wide attention from various engineering field. Generally, the existing approaches proposed in this area can be classified into major categories like, static identification and dynamic identification methods using static and dynamic test data respectively.

In static based damage indicator, an efficient indication based on the change of static strain energy is there to accurately quote the flowed elements of a damaged structure using a static analysis. The static analysis is a tool to determine the nodal displacement and internal forces of a structure subjected to static loads [3, 4]. It has the mathematical form of

$$F = K \times d \tag{1}$$

Where F is the vector of nodal loads, K is the total stiffness matrix of the structure and d is nodal displacement vector. Due to nodal displacements, strain energy is stored in each element of the structure. The strain energy of a structure due to static loads is termed here as static strain energy and can be considered as a valuable parameter for damage identification.

The dynamic identification methods is more advantageous than the static one. Among the dynamic data, the modal analysis information of a structure such as the natural frequencies, and mode shapes has been widely used for damage detection. Any damages in the structure will alter its modal parameters or the dynamic characteristics such as natural frequency, mode shape, and damping value. The reduction in stiffness is associated with decrease in natural frequencies and changes in corresponding mode shapes. The damages, reduce the stiffness of the structure, and increase the damping value. Considerable amount of researches has been done in obtaining the relationship between this modal parameters damage level and the damage location. Normalized Modal Difference (NMD)[11], Modal Assurance Criterion (MAC)[13], Co-ordinate Modal Assurance Criteria (COMAC) and Direct Natural Frequency Correlation are used as damage identification techniques to identify the damaged structure and the intensity of damage. In order to identify the locations of damaged elements Curvature Damage Factor (CDF) based on curvature mode shape was used [7, 10]. Neural Network was introduced in the final stage to determine the intensity of multiple structural damages. Numerical results show the high efficiency of the proposed method for accurately identifying, locating, and extent of multiple structural damages.

II. MATHAMATICAL MODELLING

A. Modelling of Beam

The selection of mathematical model to simulate the response of a structure is very important task in any analysis.

The Finite Elemental Method (FEM) discritize the structure into a discrete number of elements from which an approximate numerical solution is obtained. With the easy of simulating the mathematical model in FEM on personal computer, this approach provides an accurate solution for many structural analysis problems. The accuracy of result depends on the selection of suitable elements with the appropriate material characteristics modeling.

In this paper free free beam was modeled using the FEM with the commercial software package NASTRAN. In the free free beam both end nodes where free of six degree of freedom. The beam is having 24 elements and 25 nodes which satisfy the convergence as shown in Figure.1.

The material property assigned for the free free beam are given in Table 1

TABLE 1 Member property of the mathematical model

Member	Beam
Element type	1D Bar Element
Material	Steel
Length	1 m
Width	0.04 m
Depth	0.04 m
Poisson's ratio	0.3
Mass density	7850 kg/m³
Modulus of elasticity	200GPa

B. *Modelling of Damages*

There are a number of approaches to model damage in a mathematical model. Although the geometry of the damage can be very complicated, the condition is that for lower frequency vibration only an effective reduction in stiffness is required. Thus for comparison, a simple model of a damage is required. Damage can be introduced into the mathematical model by altering the material property (that is Poisson's ratio, bulk density, and modulus of elasticity). In this study modulus of elasticity has been altered by a percentage variation of -30 to +30 % with the help of Latin Hypercube sampling in MATLAB software [1, 2, 11].

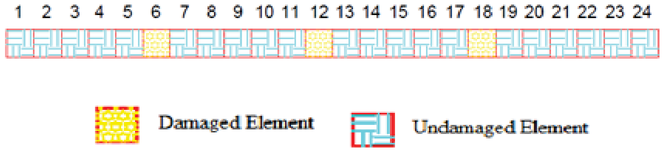

Fig.1. Damage location of free free beam

III. LOCATING THE DAMAGED STRUCTURE

In order to find out the damaged structure and intensity of damage, the mode shape and frequency of the damaged structure with a healthy structure can be compared. Damage in a structure will alter the dynamic parameters. For the calculation of intensity, the percentage variation of the mode shape and frequency can be find out by correlating the healthy and damaged structure with the help of Modal Assurance Criteria (MAC), Co-ordinate Modal Assurance Criteria (COMAC), Normalized Modal Difference (NMD) and Direct Natural Frequency Correlation.

A. *Modal Assurance Criteria*

The Modal Assurance criterion is a statistic indicator and degree of consistency between mode shapes. It is a statistical indicator that is more sensitive to large difference and relatively insensitive to small differences in the mode shapes. The Modal Assurance Criterion value is bounded between 0 & 1, with 1 indicating full consistent mode shape and a value near 0 indicates that the modes are not consistent. Generally it is found that a value above 0.9 should be attained for well correlated modes and value less than 0.1 for uncorrelated modes.

$$\mathrm{MAC} = \frac{\left|\{\varphi_A\}_r^T\{\varphi_x\}_q\right|^2}{\left(\{\varphi_A\}_r^T\{\varphi_A\}_r\right)\left(\{\varphi_x\}_q^T\{\varphi_x\}_q\right)} \quad (2)$$

Where $\{\varphi_A\}$ and $\{\varphi_x\}$ are the normalized scalar product of the two set of vectors. The resulting scalars are arranged into the MAC matrix [6, 8].

B. *Co-ordinate modal assurance criterion*

Co-ordinate Modal Assurance Criterion (COMAC) is an extension of Modal Assurance Criterion (MAC) and is calculated over a set of modal pairs, analytical versus analytical, experimental versus experimental, or experimental versus analytical. The two eigen vectors in each mode pair represent the same eigen vectors, or the mode vectors, but the set of mode pairs represent all modes of interest in a given eigen value range. The COMAC value is obtained by comparing two sets of modes corresponding to each (measurement) degree – of- freedom [13]. The COMAC value is calculated by the expression given below:

$$\mathrm{COMAC} = \frac{\sum_{r=1}^{L}\left|\Psi_{qr}\,\emptyset_{qr}\right|^2}{\sum_{r=1}^{L}\Psi_{qr}\Psi_{qr}^{*}\;\sum_{r=1}^{L}\Psi_{qr}\;\Psi_{qr}^{*}} \quad (3)$$

Where Ψ_{qr} Modal coefficient for degree of freedom q, mode r

C. *Normalized Modal Difference*

NMD is a close estimate of the average difference between the components of both vectors Φ_{aj} and Φ_{ej}. The NMD between experimental { Φ_{ej}} and analytical{ Φ_{aj}} mode shape is defined as:

$$NMD_j = \sqrt{\frac{1 - MAC_j}{MAC_j}} \quad (4)$$

In practice, the NMD is much more sensitive to mode shape differences than the MAC [11]

D. *Direct Natural Frequency Correlation*

The most common and simplest method to correlate two modal models is the direct comparison of the natural frequency. Natural frequency of a structure is a function of mass and stiffness of the structure member. Any damage occurred in a structure reduces the stiffness whereas the mass of the structure members remains the same resulting in the loss of the natural frequency of the structure. Thus a loss in a natural frequency of the structure can be used as a tool to indicate the damage in the structure. Here the natural frequency of the healthy and damaged structure is compared. The percentage difference can be defined as shown in equation given below.

$$\varepsilon_{ff} = \frac{f_{rdj} - f_{rhj}}{f_{rdj}} \times 100 \qquad (5)$$

Where fr_{dj} *and* fr_{hJ} are the frequencies corresponding to damaged and healthy structures respectively [4, 12].

IV. LOCATING THE DAMAGES BY USING MODE SHAPE CURVATURE AND CURVATURE DAMAGE FACTOR

Curvature Damage Factor based on curvature mode shape was used as a damage locating tool to effectively locate single and multiple damages in a structure. In the damaged location the stiffness of the element reduces and at that portion, the amplitude of vibration increases. By comparing the damaged structure with an undamaged structure effectively the damage can be effectively located.

A. Mode Shape Curvature method

It is likely that damage indicators based on derivatives of the mode shape will amplify the localized damages in a structure .The curvature mode shape has emerged as one of the best way to amplify the effect of the damage on the mode shape. The curvature mode shapes are based on flexural stiffness of the beam cross section. Based on beam theory the curvature at a point in the beam is given by

$$V'' = M / (E^{b}_{xx} I_{yy}) \qquad (6)$$

Where M is the bending moment at the section and $(E^{b}_{xx} I_{yy})$ is the flexural stiffness of the beam.

The presence of damage in a beam at a given location reduces the flexural stiffness of the beam and hence increases the magnitude of curvature at the damaged location. Typically damages occurred due to impact and are likely to be localized at some point in the structure. The changes in curvature are local in nature and can be used to find the damage location in the beam. To obtain curvature mode shape of a damaged beam finite element analysis is done to get the displacement mode shape. Then using displacement mode shape, curvature mode shapes are obtained numerically by a central difference approximation as:

$$V_{i,j} = \frac{\emptyset_{(i+j),j} - 2\emptyset_{i,j} + \Phi_{(i-1),j}}{h_e} \qquad (7)$$

Where $V_{i,j}$ represents curvature mode shape, subscript i represent the node number and subscript j represents the mode number. Also h_e represents the finite element length and $\Phi_{i,j}$ represents the mass normalized displacement mode shape for the i^{th} mode shape.

Absolute difference in curvature mode shape between damaged and undamaged structure is obtained as;

$$\Delta v_{i,j} = v_{i,j}^{(d)} - v_{i,j}^{(u)} \qquad (8)$$

The Curvature Damage Factor (CDF) is obtained by averaging the first few curvature mode shape. In general CDF of i^{th} node is obtained by considering the first n curvature mode shape and is given as;

$$CDF = \frac{1}{N} \sum_{j=1}^{N} \Delta v_{i,j} \qquad (9)$$

The CDF at each node is obtained by considering the first five curvature mode shape. With increase in damage density, the peak magnitude of CDF at the damage location also increases and hence indicates the extent of damage [5, 7, 10].

V. QUANTIFYING THE INTENSITY OF DAMAGE BY USING NEURAL NETWORK

In the third stage after localizing the damage, the intensity of damage (joint stiffness) in each particular damage locations has to be calculated. For this Neural Networking method is adopted as a tool for determining the joint stiffness of each damaged elements.

A. Neural Network

An artificial Neural Network is an information processing paradigm that is inspired by biological nerve system. It is composed of a large number of highly interconnected processing elements called nerves. A Neural Network is configured for a special application, such as pattern recognition or data classification. The use of Neural Network is that it's ability to derive meaning from complicated or imprecise data. The main advantage is that it can extract patterns and detect trends that are too complex to be noticed by either humans or other computer techniques. Conventional computers use an algorithmic approach, but Neural Network works similar to human brain and learns by examples. The layers in a neural network is shown in the Figure.2

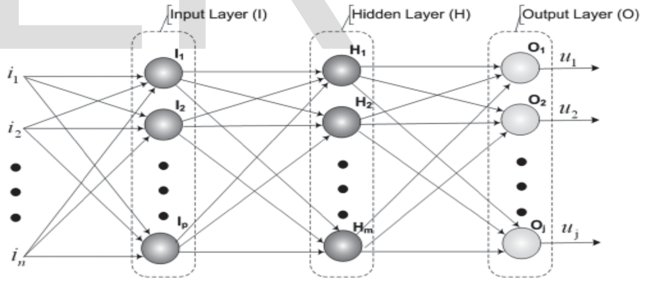

Fig.2. Layers in a neural network

Neural network systems allow for the correlation of complex nonlinear systems without requiring explicit knowledge of the functional relationship that exists between the input and output variables of the system. Further, algorithms with neural network techniques are inherently stable for the calibration of nonlinear data involving more number of independent parameters [9, 11].

In this paper Neural Network is used to represent the mapping between frequency domain data and modal parameters. Once trained, the Neural Network quickly yields accurate estimation of the modal parameters based on the frequent domain response of the structure. As the process of estimating the modal parameter is fast, this technique can be used to adjust the control law acting on the structure in real time as long as parameter variations are slow enough to allow for the updating of system. Hence the inference is that frequency based damage detection with the help of Neural Network by frequency comparison of healthy to the damaged structure will effectively quantify the intensity of damage.

VI. RESULTS AND DISCUTIONS

A. Modal Assurance Criteria

The MAC values of eight mode shapes for free free beam is shown in Table 2**.**

TABLE 2 MAC values of 8 mode shapes for free free beam

Mode No	**MAC**
1	1
2	1
3	1
4	0.999
5	0.998
6	0.999
7	0.999
8	0.995

B. Co-ordinate modal assurance criterion

The COMAC value of the degrees of freedom of nodes for free free beam is presented in Table 3**.**

TABLE 3 COMAC with respect to the degrees of freedom of the nodes for free free beam

DOF	**COMAC**
3	0.99951
9	0.99991
15	0.99906
21	0.99925
27	0.99983
33	0.99902
9	0.99875
45	0.99938
51	0.99953
57	0.99897
63	0.99907
69	0.99981
75	0.9938
81	0.99859
87	0.99587
93	0.99601
99	0.99882
105	0.99384
111	0.99988
117	0.99959
123	0.99967
129	0.99972
135	0.99976
141	0.99991
147	0.9998

C. Normalized Modal Difference

The NMD values of eight mode shapes for free free beam is given in Table 4**.**

TABLE 4 NMD of 8 mode shape for free free beam

Mode No	**NMD**
1	0.006
2	0.012
3	0.016
4	0.034
5	0.039
6	0.037
7	0.032
8	0.072

D. Direct Natural Frequency Correlation

The Direct Natural Frequency correlation of eight mode shapes for free free beam is tabulated in Table 5**.**

TABLE 5 Direct natural frequency correlation for free free beam

Mode no	**Analysis freq**	**Exp freq**	**% Error**
1	972.1	967.98	-0.4251
2	2331.07	2327.13	-0.169
3	3890.33	3884.43	-0.152
4	5463.21	5450.9	-0.2259
5	6993.55	6976.82	-0.2397
6	8461.22	8409.31	-0.6172
7	9862.25	9860.69	-0.0158
8	11194.17	11195.27	0.0098

E. Difference in Curvature Mode Shape

Locating the damaged positions using Difference in Curvature Mode Shape 1, 2, and 10 are shown in Figure. 3, 4, and 5.

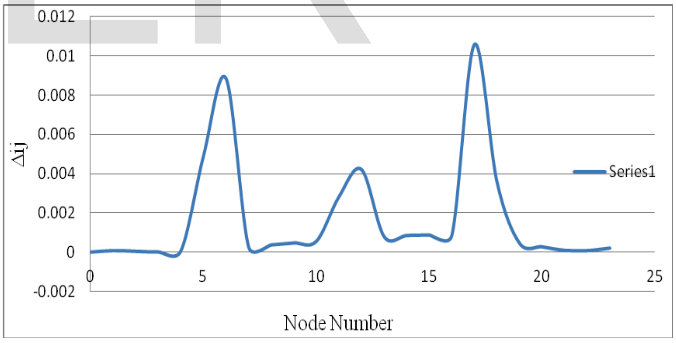

Fig.3. Difference in curvature mode shape 1 for free free beam

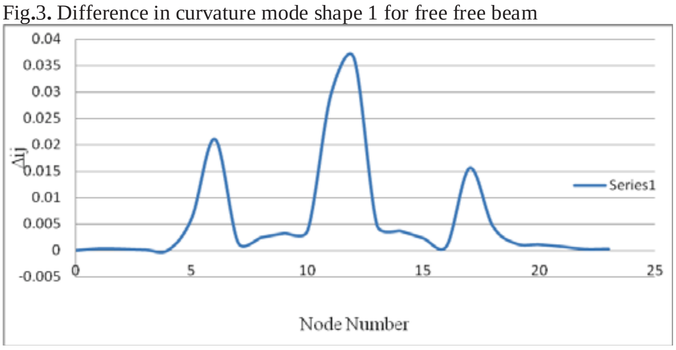

Fig.**4.** Difference in curvature mode shape 2 for free free beam

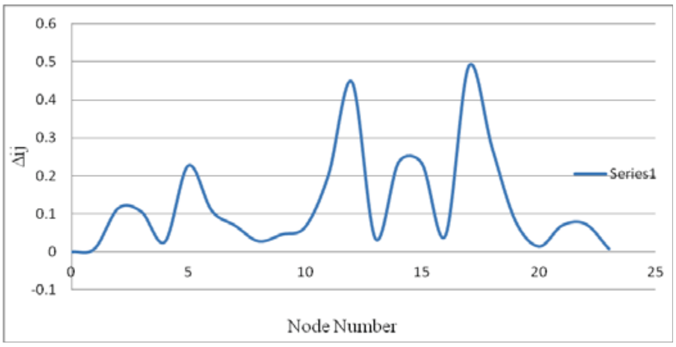

Fig.**5.** Difference in curvature mode shape 10 for free free beam

F. Curvature Damage Factor

Locating the damaged positions using Curvature Damage Factor for Mode Shape 1, 2, and 10 are shown in Figure. 6, 7, and 8.

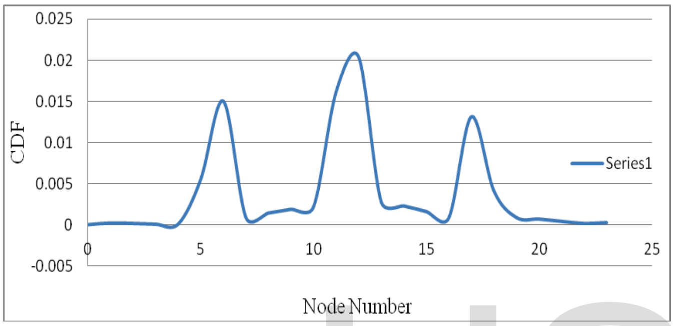

Fig.**6.** Curvature damage factor 1 for free free beam

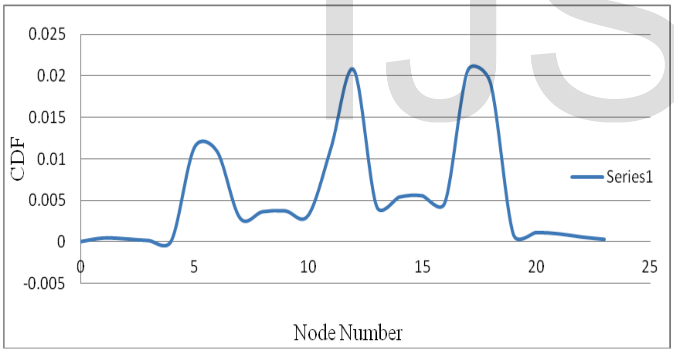

Fig.**7.** Curvature damage factor 2 for free free beam

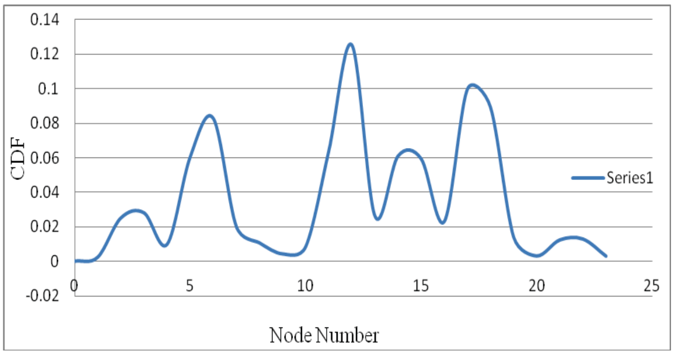

Fig.**8.** Curvature damage factor 10 for free free beam

G. Neural Network

The results obtained after training and validating the Neural Network is tabulated in the table 6.

TABLE 6 validation results from neural network

	Joint Stiffness in Elements (GPa)			**Neural Network Results in Elements (GPa)**		
	6	12	18	6	12	18
Validation/ Analysis	179	180	227	178	180	227
Validation/ Analysis	200	211	171	200	211	170
Test	222	190	222	222	190	222
Test	174	203	179	174	203	179

H. Summary

The damage detection is a three stage process which includes correlating, locating, and quantifying. That is, correlating the healthy and damaged structure, locating the damage and quantifying the intensity of damage at an element level.

In the first stage, for the identification of damaged structure, correlate the healthy structure with an unhealthy structure and thereby obtain the intensity of the damage. The percentage differences in the dynamic parameters are noted down to evaluate the intensity of damage. For this, Modal Assurance Criterion (MAC), Co-ordinate Modal Assurance Criterion (COMAC), Normalized Modal Difference (NMD), and Direct Natural Frequency Correlation are used. The MAC is one of the popular tools for the quantitative comparison of modal vectors and a statistical indicator. This least squares based form of linear regression analysis yields an indicator that is more sensitive to the largest difference between comparative values and results in MAC that is insensitive to small changes or small magnitudes. Coming to COMAC it is an extension of MAC which will give the displacements at the nodes corresponding to the degrees of freedom on the each individual node. It is also more sensitive to largest difference between comparative values and insensitive to smaller magnitudes. NMD is a closer estimate of the average difference between the components of both vectors of healthy and damaged structure. In practice the NMD is much more sensitive to mode shape difference than the MAC. The most common and simplest approach to correlate two model modals is the direct comparison of the natural frequencies. A percentage difference can be obtained most effectively by using this method.

In the second stage, locate the damaged element can be located with the help of Curvature Damage Factor (CDF) based on Curvature Mode Shape. In Mode Shape Curvature method, the Curvature Damage Factor (CDF) is obtained by averaging the first few Curvature Mode shapes between damaged and undamaged beams. The location of the damage is identified by a sudden change in CDF in Mode Shape Curvature method**.** It is capable of successfully locating single and multiple damaged sights.

Coming to the final stage, quantifying the intensity of damage at an element level for validate the damage intensity of each damaged element**.** Neural Network is used as a tool to calculate the intensity of damage**.** The mathematical model of the structure based on dynamic parameters train the neural network to take input as frequency and output as modulus of elasticity. Based on the trained data the Neural Network which predict the value of modulus of elasticity corresponding to the Frequency Response Function (FRF) has been built. The first five mode shapes and frequencies were taken for evaluation**.** So the inference is that frequency based damage detection with the help of Neural Network will effectively quantify the intensity of damage.

VII. CONCLUSIONS

A three-stage method for detection, locating, and quantifying using dynamic data has been presented. First MAC, COMAC, NMD and Direct Natural Frequency Correlations were used to find out the damaged structure. Then Curvature Mode Shapes of the damaged beam and CDF is used as a damage indicator to identify the damage location at an element level. Finally Neural Network is used as quantifying tool to determine the damage extent. If 'n' number of element have been selected as suspected damage elements, this three stage method can effectively locate and quantity the extent of damage of each individual elements. Thus the proposed three stage method is more effective. The numerical results show the high efficiency of the proposed method for accurately identifying, locating, and extent of multiple structural damages.

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